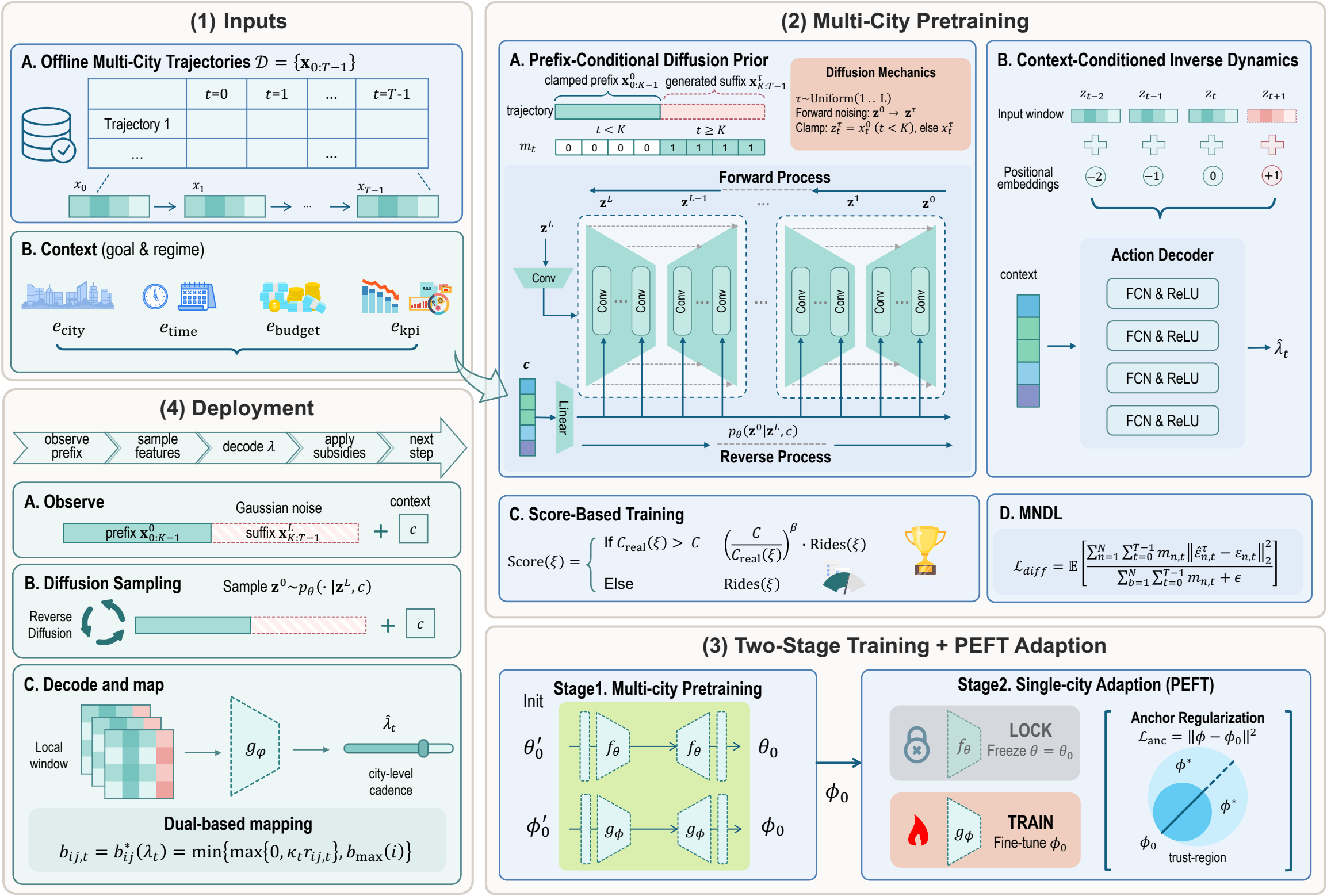
(1) Inputs
A. Offline Multi-City Trajectories $\mathcal{D} = \{\mathbf{x}_{0:T-1}\}$
t=0
t=1
...
t=T-1
Trajectory 1
B. Context (goal & regime)
e_{city}
e_{time}
e_{budget}
e_{kpi}
(2) Multi-City Pretraining
A. Prefix-Conditional Diffusion Prior
clamped prefix $\mathbf{x}^0_{0:K-1}$
generated suffix $\mathbf{x}^\tau_{K:T-1}$
trajectory
$t < K$
$t \geq K$
m_t
Diffusion Mechanics
$\tau \sim \text{Uniform}(1 .. L)$
Forward noising: $\mathbf{z}^0 \rightarrow \mathbf{z}^\tau$
Clamp: $z^\tau_t = x^0_t$ $(t < K)$, else x^τ_t
Forward Process
Conv
Linear
$p_\theta(\mathbf{z}^0|\mathbf{z}^L, c)$
Reverse Process
B. Context-Conditioned Inverse Dynamics
Input window
Positional embeddings
context
Action Decoder
FCN & ReLU
FCN & ReLU
FCN & ReLU
FCN & ReLU
C. Score-Based Training
If $C_{\text{real}}(\xi) > C$
Else
Rides(ξ)
D. MNDL
$\mathcal{L}_{diff} = \mathbb{E}\left[\frac{\sum_{n=1}^{N}\sum_{t=0}^{T-1} m_{n,t}\left\|\hat{\varepsilon}^\tau_{n,t} - \varepsilon_{n,t}\right\|_2^2}{\sum_{b=1}^{N}\sum_{t=0}^{T-1} m_{n,t} + \epsilon}\right]$
(3) Two-Stage Training + PEFT Adaption
Stage1. Multi-city Pretraining
Init
θ'_0
ϕ'_0
θ_0
ϕ_0
Stage2. Single-city Adaption (PEFT)
LOCK
Freeze $\theta = \theta_0$
TRAIN
Fine-tune ϕ_0
Anchor Regularization
$\mathcal{L}_{\text{anc}} = \|\phi - \phi_0\|^2$
trust-region
(4) Deployment
observe prefix
sample features
decode λ
apply subsidies
next step
A. Observe
Gaussian noise
prefix $\mathbf{x}^0_{0:K-1}$
suffix $\mathbf{x}^L_{K:T-1}$
context
B. Diffusion Sampling
Sample $\mathbf{z}^0 \sim p_\theta(\cdot|\mathbf{z}^L, c)$
Reverse Diffusion
C. Decode and map
Local window
city-level cadence
Dual-based mapping
$b_{ij,t} = b^*_{ij}(\lambda_t) = \min\{\max\{0, \kappa_t r_{ij,t}\}, b_{\max}(i)\}$